Efficient Implementation of Simplified Model Predictive Control

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One of the recent digital control algorithms is known as the simplified model predictive control (SMPC) algorithm. Originally proposed by Tu and Tsing in 1979, it received the attention of researchers in the latter half of 1980s (Arulalan and Deshpande, 1986, 1987; Vaidya and Deshpande, 1988). The SMPC algorithm has many attractive features, such as zero offset, a single tunable parameter, some dead time compensation, and others (Vaidya and Deshpande, 1988).

One disadvantage of SMPC compared with PID (proportional-integral-derivative) control is the computational effort; it involves $N + 2$ multiplications (and relatively inexpensive N additions) where N is the number of sampling periods in the settling time, Eq. 2. This note describes a simple yet effective implementation of SMPC that significantly reduces the required computational effort. Furthermore, the proposed modification improves the performance of SMPC for many applications of practical importance.

SMPC Algorithm

The block diagram of a simplified sampled data control system is shown in Figure 1. The z-transform representation is assumed throughout. The SMPC algorithm is based on the premise that it should always be possible to design a control algorithm that will yield a closed-loop set point response that is at least as good as the normalized open-loop response. See Vaidya and Deshpande (1988) for a detailed derivation of SMPC. The final equations are

$$D(z) = \alpha \Bigg/ \left[1 - \left(\sum_{i=1}^{N} h_i z^{-i} \right) \Bigg/ K_p \right] \quad (1)$$

$$M_n = \alpha E_n + (1/K_p)(h_1 M_{n-1} + h_2 M_{n-2} + \cdots + h_N M_{n-N}) \quad (2)$$

The tunable parameter, α, is selected based on minimizing a suitable performance criterion such as ISE (integral of square of error), IAE (integral of absolute error), or ITAE (integral of time × absolute error).

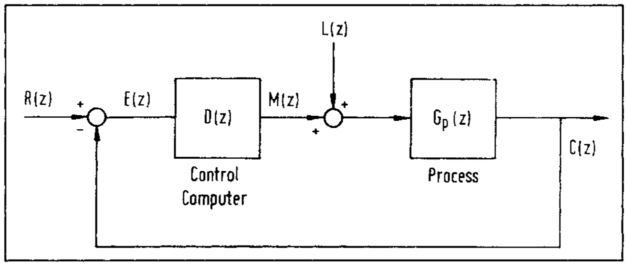

Figure 1. Typical sampled data control system.

Equation 2, which is in the time domain, gives the manipulated variable at the nth sampling instant, and involves mainly $N + 2$ multiplications. The value of N depends on settling time and sampling period. For first order plus dead time (FODT) processes, the recommended sampling period is in the range of 1/10 to 1/20 of the time constant. In this study, the sampling period is taken as 1/15 of the time constant.

Settling (or response) time, t_f, is the time required for the open-loop process response (to a step change in the input) to reach within a certain fraction f of the steady state value. For example, in the case of an FODT process with zero dead time and unit time constant, $t_{0.01}$ is 4.605. The selection of f for SMPC implementation, although important, has not received much attention. By considering the closed-loop response with $D(z)$ as given by Eq. 1, it can be shown that, for SMPC, f should be as small as possible for minimizing offset. Since a small f results in a large N, a practical value for f seems to be 0.001.

Efficient Implementation of SMPC

We propose a simple modification of replacing K_p by $\Sigma_{i=1}^{N} h_i$ in Eqs. 1 and 2 for implementing SMPC. This ensures zero offset irrespective of the value selected for f. For example, the closed-loop response of the feedback loop in Figure 1 to a unit

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Table 1. Number of Sampling Periods in the Settling Time

θ	N for $f =$ 0.001	0.01	0.05	0.10	0.20
0.1	106	71	47	37	26
1.0	119	85	60	50	40
5.0	179	145	120	110	100

Sampling period = Time Constant/15

step disturbance in set point is

$$C(z) = [1/(1 - z^{-1})]\{D(z)G_p(z)/[1 + D(z)G_p(z)]\} \tag{3}$$

where

$$D(z) = \alpha \Bigg/ \left[1 - \left(\sum_{i=1}^{N} h_i z^{-i}\right) \Bigg/ \left(\sum_{i=1}^{N} h_i\right)\right] \tag{4}$$

Using the final value theorem of z-transforms, the steady-state value is given by the limit of $(1 - z^{-1})\ C(z)$ as $z \to 1$. Substituting Eqs. 3 and 4 in this, and taking the limit, the final value is

$$\alpha K_p/\{1 - (\Sigma h_i)/(\Sigma h_i) + \alpha K_p\} = 1 \tag{5}$$

Hence there is no offset for set point disturbances.

In the above proof, no assumption was made on N; it could be based on any f and selected sampling period. Hence, the proposed implementation can be used with a larger f, and consequently a smaller t_f and N. The variation of N with f for some typical cases is shown in Table 1. Note that optimum α depends on f, and hence it has to be determined for the chosen f.

Hereafter, SMPC based on Eq. 1 with $f = 0.001$ will be referred to as the standard SMPC, while the proposed implementation of SMPC, Eq. 4, will be called the efficient SMPC. There is another possible advantage of the proposed modification. Referring to Eq. 2, reduction of N effectively gives more weighting to the manipulated variable at recent sampling instants (M_{n-1}, M_{n-2}, etc.), and the manipulated variable at some oldest sampling instants is eliminated. This, as will be shown later, improves the performance of the efficient SMPC for load disturbances, which are frequently encountered in process control.

Evaluation of the Efficient SMPC

The efficient SMPC is evaluated for a range of f values (0.01, 0.05, 0.1, 0.2) through simulation. The process is assumed to be FODT with $K_p = 1$ and unit time constant. A series of values for dead time, θ (0.05, 0.1, 0.4, 1.0, 2.0, 5.0), covering small to large dead time situations, both load and set point disturbances, and all the three optimization criteria—ITAE, IAE, and ISE—are tried. α is optimized for the selected f, type of disturbance and optimization criterion. The performance of the efficient SMPC for unit step disturbance in load and set point is summarized in Tables 2 and 3, respectively. The performance of the standard SMPC is also shown in these tables for the purpose of comparison.

The results in Table 2 indicate that, for load disturbance, the efficient SMPC is generally better than the standard SMPC, over the range of f studied, when dead time is small, about 0.4 or less. For larger dead time cases also, it is comparable to the standard SMPC for smaller f; for ITAE and IAE, f should be about 0.05 or less while for ISE f can be as large as 0.1. Table 3 shows that the efficient SMPC is either comparable or inferior to the standard for set point disturbance. Both are comparable for IAE and ISE provided f is about 0.05 or less.

The profiles of response and manipulated variable for the efficient SMPC, are similar to those for the standard. As an example, these profiles for the standard and efficient (for $f = 0.1$) SMPC are shown in Figure 2 for the case of $\theta = 0.4$,

Table 2. Performance of SMPC for Load Disturbance

θ	Standard SMPC, $f = 0.001$	Efficient SMPC, $f =$ 0.01	0.05	0.10	0.20
		ITAE Minimization			
0.05	0.0594	0.0533	0.0422	0.0348	0.0255
0.1	0.1044	0.0946	0.0782	0.0707	0.0544
0.4	0.6202	0.5928	0.5611	0.5891	0.5759
1.0	3.232	3.165	3.180	3.313	4.563
2.0	11.26	11.10	12.28	13.50	15.45
5.0	54.09	54.43	56.85	59.06	62.73
		IAE Minimization			
0.05	0.0569	0.0552	0.0510	0.0476	0.0408
0.1	0.0961	0.0935	0.0871	0.0839	0.0758
0.4	0.4184	0.4128	0.4012	0.4078	0.4011
1.0	1.233	1.232	1.226	1.243	1.438
2.0	2.565	2.555	2.659	2.789	2.996
5.0	5.942	5.965	6.089	6.202	6.389
		ISE Minimization			
0.05	0.0024	0.0023	0.0022	0.0021	0.0020
0.1	0.0068	0.0068	0.0065	0.0063	0.0058
0.4	0.1017	0.1013	0.0999	0.0991	0.0986
1.0	0.5393	0.5391	0.5380	0.5389	0.5641
2.0	1.535	1.535	1.543	1.564	1.620
5.0	4.656	4.657	4.670	4.689	4.728

Table 3. Performance of SMPC for Set Point Disturbance

θ	Standard SMPC, $f = 0.001$	Efficient SMPC, $f =$ 0.01	0.05	0.10	0.20
		ITAE Minimization			
0.05	0.0164	0.0308	0.0494	0.0584	0.0677
0.1	0.0405	0.0637	0.0954	0.1116	0.1307
0.4	0.3517	0.4158	0.5550	0.6678	0.8188
1.0	1.469	1.625	2.025	2.380	3.294
2.0	4.260	4.521	5.335	6.071	7.231
5.0	18.31	18.81	20.35	21.68	23.76
		IAE Minimization			
0.05	0.1360	0.1408	0.1526	0.1632	0.1779
0.1	0.2326	0.2395	0.2563	0.2690	0.2889
0.4	0.7464	0.7591	0.8048	0.8556	0.9378
1.0	1.594	1.614	1.707	1.8039	1.984
2.0	2.795	2.825	2.944	3.053	3.241
5.0	5.963	5.999	6.115	6.217	6.378
		ISE Minimization			
0.05	0.0841	0.0841	0.0845	0.0846	0.0858
0.1	0.1597	0.1598	0.1602	0.1610	0.1638
0.4	0.5687	0.5691	0.5720	0.5774	0.5948
1.0	1.280	1.281	1.288	1.3021	1.350
2.0	2.371	2.372	2.382	2.402	2.450
5.0	5.447	5.448	5.457	5.472	5.505

load disturbance, and ITAE minimization. Optimum α for the efficient SMPC differs from that for the standard; the difference is either small or as much as 50%, depending on f, disturbance type, and optimization criterion.

In many process control applications, dead time is less than the time constant and frequent disturbances are in load. For these situations, the efficient SMPC with $f =$, say, 0.1 provides better performance than the standard. The results in Tables 2 and 3 indicate that there is probably an optimum f for each θ and the chosen performance criterion.

The main advantage of the efficient SMPC is the reduction in N and consequently the computational effort during each sampling period, which is important when a single microprocessor has to control dozens of control loops. Also, there is a corresponding reduction in the number of past data (M_n's) to be stored. The extent of change in N can be seen from the tabulated values in Table 1. For $\theta = 1$, N decreases by a factor of 2 when f is increased from 0.001 to 0.1.

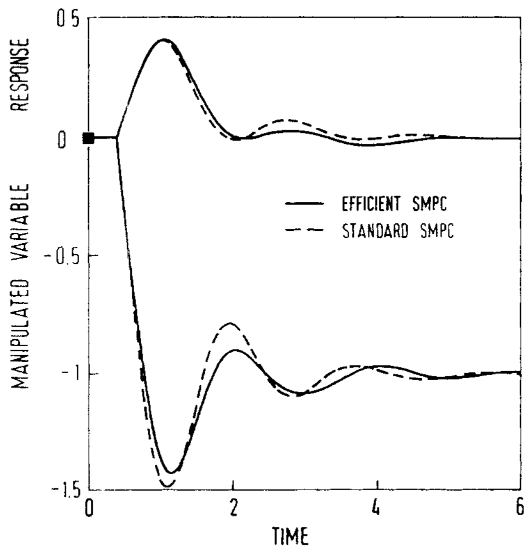

Figure 2. Profiles of response and manipulated variable for standard SMPC and efficient SMPC.
($f = 0.1$; $\theta = 0.4$; load disturbance and ITAE minimization)

Notation

C = controlled variable
D = feedback controller
E = error
f = fraction for defining settling time
G_p = process transfer function
h_i = impulse response coefficients
K_p = steady state gain of process
L = disturbance variable
M = manipulated variable
N = number of sampling periods in settling time
R = set point
T = sampling period
t_f = settling time
z = z-transform operator

Greek letters

α = SMPC tuning parameter
θ = process dead time

Literature Cited

Arulalan, G. R., and P. B. Deshpande, "New Algorithm for Multivariable Control," *Hydro. Process.*, 51–54 (June 1986).

———, "Simplified Model Predictive Control," *Ind. Eng. Chem. Res.*, **26**, 347 (1987).

Tu, F. C. Y., and J. Y. H. Tsing, "Synthesizing a Digital Algorithm for Optimized Control," *InTech*, 52–55 (May 1979).

Vaidya, C. M., and P. B. Deshpande, "Single-Loop Simplified Model Predictive Control," *Hydro. Process.*, 53–57 (June, 1988).

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